Nonlinear Optics

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## The Laws of Linear Optics

- Optical properties are independent on light intensity
  \[ n, \alpha, \ldots \neq f(|E|^2) \]

- Superposition principle
  \[ F(\sum a_i x_i) = \sum a_i F(x_i) \]

- Radiation does not change its frequency
  \[ \omega_{In} = \omega_{Out} \]

- No radiation interactions
Nonlinear Optics

Since invention of laser in 1960:

The Laws of Linear Optics

• Optical properties are independent on light intensity $n, \alpha, \ldots \neq f(|\mathbf{E}|^2)$

• Superposition principle

$$F(\sum a_i x_i) = \sum a_i F(x_i)$$

• Radiation does not change its frequency

$$\omega_{In} = \omega_{Out}$$

• No radiation interactions
\[ E_{\text{ext}} = 0 \]

... positive charge

... negative charge

• ... positive charge

○ ... negative charge
$E_{\text{ext}} = 0$

- ... positive charge
- ... negative charge

$E_{\text{ext}}$
Response of a Matter

Principle of Causality

\[ P(r, t) = \varepsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi(r - r', t - t') E(r', t') \, dr' \, dt' \]

Dielectric Polarization

\[ P(\omega) = \varepsilon_0 \chi^{(1)} E(\omega) + \varepsilon_0 \chi^{(2)} E^2(\omega) + \varepsilon_0 \chi^{(3)} E^3(\omega) + \cdots \]

Property of matter NOT of a radiation!
Why One Needs a Laser?

\[
P(\omega) = \varepsilon_0 \chi^{(1)}(\omega)E(\omega) + \varepsilon_0 \chi^{(2)}(\omega)E^2(\omega) + \varepsilon_0 \chi^{(3)}(\omega)E^3(\omega) + \cdots
\]

- \( \chi^{(1)} > \chi^{(2)} > \chi^{(3)} > \cdots \)
- \( \chi^{(2)} \approx \chi^{(1)} \iff E_{\text{Field}} \approx E_{\text{Atom}} (\sim 10^{11} \text{ V/m}) \)

\[ \implies \]
- Linear processes are stronger than nonlinear ones
- Nonlinear process of \( n^{\text{th}} \) order is stronger than one of \( (n + 1)^{\text{th}} \) order
\[ P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} EE + \varepsilon_0 \chi^{(3)} EEE + \cdots \]

\[ P = P_L + P_{NL}; \]

\[ P_{NL} = \varepsilon_0 \chi^{(2)} EE \]
Large Variety of Processes

\[ E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + \text{c.c.} \]

\[ P_{NL}(t) = \varepsilon_0 \chi^{(2)} EE \]
\[ = 2\varepsilon_0 \chi^{(2)} (E_1 E_1^* + E_2 E_2^*) + \]
\[ + \varepsilon_0 \chi^{(2)} \left( E_1^2 e^{-i2\omega_1 t} + E_2^2 e^{-i2\omega_2 t} \right) + \text{c.c.} + \]
\[ + 2\varepsilon_0 \chi^{(2)} \left( E_1 E_2 e^{-i(\omega_1+\omega_2)t} + E_1^* E_2^* e^{-i(\omega_1-\omega_2)t} \right) + \text{c.c.} \]

- Second-harmonic generation (SHG)
- **Sum-frequency generation (SFG)**
- Difference-frequency generation (DFG)
Sum-Frequency Generation

Three-Wave mixing

Second-harmonic generation: \[ \omega_1 = \omega_2 \implies \omega_2 = 2\omega_1 \]
Conservation Laws

Energy:

\[ \hbar \omega_3 = \hbar \omega_1 + \hbar \omega_2 \]

Momentum:

\[ \hbar k_3 = \hbar k_1 + \hbar k_2 \]

Collinear:

\[ k_1 \quad k_2 \quad k_3 \]

Non-Collinear:

\[ k_1 \quad k_2 \quad k_3 \]
Momentum Conservation Law

Phase-Matching Condition:

\[ \Delta k = k_3 - k_1 - k_2 = 0 \]

Efficiency:

\[ I = I_{\text{max}} \cdot \text{sinc}^2 \left( \frac{\Delta k L}{2} \right) \]
Phase-Matching Condition

Second-harmonic generation in collinear geometry:

Frequency: \( \omega_3 = \omega_1 + \omega_2 \rightarrow \omega_2 = 2\omega_1 \)

Momentum: \( k_3 = k_1 + k_2 \rightarrow k_2 = 2k_1 \)

Phase-Matching Condition:

\[
\Delta k = k_2 - 2k_1; \quad k_j = \frac{\omega_j n_j(\omega_j)}{c}
\]

\[
\Delta k = 0 \iff n_2(\omega_2) = n_1(\omega_1)
\]
Birefringence

- Optical property of material (anisotropy)
- Refractive index depends on
  - Polarization
  - Propagation direction
Indicatrix
Indicatrix
Birefringence - Phase-Matching

\[ n_{\text{Ordinary}} \]
\[ n_{\text{Extra-ordinary}} \]

\[ \theta \]
Birefringence - Phase-Matching

\[ n_{\text{Ordinary}} \quad n_{\text{Extra-ordinary}} \]

\[ n(\omega_1) \quad n(\omega_2) \]
Birefringence - Phase-Matching

\[ n_{\text{Ordinary}} \]

\[ n_{\text{Extra-ordinary}} \]

\[ n(\omega_1) \]

\[ n(\omega_2) \]
KTP - Refractive Indices

![Graph showing refractive indices of KTP against wavelength (nm). The graph plots refractive index vs. wavelength for different orientations (nx, ny, nz) with distinct curves for each. The data points are marked at specific wavelengths (400, 500, 600, 700, 800 nm).]
Applications

- Generation of different frequencies
- Parametric amplification and oscillation
- Self-focusing
- Intensity autocorrelation
- ... and many others ...
See more in the lab!