Joint International Physics Summer School
Insubria University (Como) – Palacky University (Olomouc)
Optics

Spectrum ...

LA STRUTTURA DELLA LEZIONE

S. Spectrum, spectroscopy
F. Fourier series and transformation
P. Signals and short pulses
4. Spatial/k and frequency/time modes, resonators
L. Lasers and spectrum
5. Programming
SPECTRUM

Light intensity (power) dependence on wavelength (frequency) \[ \nu = \frac{c}{\lambda} = \frac{\omega}{2\pi} \]

gamma  rtg  ultraviolet  visible  infrared  radio
Spectra types

- 1 continuous (caused by continuous energy levels)
- 1 line (caused by discrete energy levels)
  - emission (light produced by the matter)
  - absorption (light absorbed by the matter)

continuous spectrum: black body emission spectrum

Planck's law – radiation of black body

\[ I = \frac{2\hbar \nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \]

black body = perfect absorber: absorbs arbitrary energy (even line spectrum), transforms it into the same (temperature dependent) spectrum (example: Edison's bulb)

application:
- source for spectral attenuation measurement
- temperature measurement

http://en.wikipedia.org/wiki/Max_Planck

Continuous spectrum: black body – [no_advertisement]'s bulb

(Industrial) bulb
E27 230V 300W

- Continuous light sources are important in the industry to avoid stroboscopic effect
- The bulb lies in a case on main choir of St. Michael's church (holy mass Mo-Fri 16:00) [advertisement]
line spectra (Hydrogen)

\[ \frac{1}{\lambda} = R \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad R = 1.097 \times 10^7 \text{ m}^{-1} \] (Rydberg)

application:
- chemical analysis
- wavelength calibration
- special – spectral line shifts
Spectrum measurement

principle = using of spectral dependence of something:
• dispersion (refractive index dependence)
• diffraction (constructive interference wavelength dependence)
• using set of [narrow] spectral filters or tunable filter
• other – Fourier spectroscopy (FT of autocorrelation function (time properties of propagation)), ...

Dispersion

\[ n_1(\lambda) \sin(\theta_1) = n_2(\lambda) \sin(\theta_2) \] (Snell)

normal dispersion: \( \frac{dn}{d\lambda} < 0 \) \( (n(\lambda_{\text{red}}) < n(\lambda_{\text{violet}})) \)
abnormal dispersion: \( \frac{dn}{d\lambda} > 0 \)

Spectrum measurement

Dispersion prism

\[ \theta_d = \theta - \alpha + \arcsin \left[ \sin \alpha \sqrt{n^2(\lambda) - \sin^2(\theta)} - \sin \theta \cos \alpha \right] \]

Dispersion on rain drops

Černoch: Spektroskopie

Černoch: BEM3
Spectrum measurement

Diffraction
- Huygen's wave principle

reflective grating

$\alpha(\lambda) = ?$
Spectrum measurement

Diffraction grating

\[ \sin \theta_q = \sin \theta_i + q \frac{\lambda}{\Lambda}, \quad q \in \mathbb{Z} \quad (\text{diffraction order}) \]

\( \Lambda \) – grating period (lines distance)

\[ \theta_q = \theta_i + q \frac{\lambda}{\Lambda}, \quad q \in \mathbb{Z}, \quad \Lambda \gg \lambda, \quad \text{paraxial} \rightarrow \text{linear} \]

Černoch: Spektroskopie

transmittive grating

1. input SMA connector, 2. slit,
3. frequency filter, 4. collimating mirror,
5. grating, 6. focussing mirror, 7. back-illuminated CCD detector, 8. higher-order diffraction blocking filter

Spectrum measurement

Fabry-Perot plane resonator – tunable interference filter

Transmittance

\[ T(\nu) = \frac{T_{\text{max}}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2\left(\frac{\pi \nu}{\nu_F}\right)} \]

\[ T_{\text{max}}(\nu) = \frac{(1 - |r_1|^2)(1 - |r_2|^2)}{(1 - |r_1||r_2|)^2} \]

Wavelength modes

\[ \lambda_q = \frac{2d}{q}, \ q \in \mathbb{Z} \]

Free spectral range

\[ \nu_F = \frac{c}{2d} \]

Finesse

\[ F = \frac{\pi \sqrt{|r_1 r_2|}}{1 - |r_1 r_2|} \]

Good resolution, requires external filter to select range
Time-developed spectrum

OPERATING PRINCIPLE

Trigger signal

Sweep electrode (where electrons are swept in the direction from top to bottom)

Streak image on phosphor screen

Optical intensity

Time

Space

Incident light

Slit

Lens

Accelerating electrode (where electrons are accelerated)

MCP (which multiplies electrons)

Photocathode (light $\rightarrow$ electrons)

Sweep circuit

Phosphor screen (electrons $\rightarrow$ light)

The intensity of the incident light can be read from the brightness of the phosphor screen, and the time and space from the position of the phosphor screen.

(Hamamatsu)

Streak camera

monochromator (spectrometer)

streak tube

CCD
Harmonic function (mathematically nice function)

\[ f(t) = A \cos(2\pi \nu_0 t + \phi) \]

- for characterization of harmonic function with given frequency \( \nu_0 \), two [real] coefficients \( (A, \phi) \) are needed
- e.g. Electrical intensity of electromagnetic field on given space in different time

Instead of sine and cosine, we are working with more friendly exponential functions

\[ \cos(2\pi \nu_0 t + \phi) = \Re \left[ e^{i(2\pi \nu_0 t + \phi)} \right] \]
Fourier series

Harmonic function and its Fourier series

\[ \frac{1}{V_0} = T \]

signal - time (or space) domain

amplitude

phase

spectrum - frequency (wavelength) domain
Fourier series

Superposition of harmonic functions

- 2.1 fs
- 1.4 fs
- 4.2 fs = least common multiplicative of 2.1 and 1.4

The phase is important.

- t
- \( \nu \)
- Amplitude
- Phase

- 650 nm
- 461 THz

- 433 nm
- 692 THz

1.4 fs

4.2 fs
Fourier series

Superposition of frequency-equidistant (synchronized) harmonic functions

$F_f$
Fourier series on a scope

harmonic generator I

≈

synchronization

harmonic generator II

oscilloscope

1st harm (cos), $\varphi=90$ deg

2nd harm, $A=300\,\text{mV}$, $\varphi=180$ deg

short pulse made from 1st harm + 2nd harm

3rd harm, $\varphi=90$ deg

triangle made from 1st harm + 3rd harm

1st harm (cos), $\varphi=90$ deg

rectangular made from 1st harm + 3rd harm
Fourier series

Decomposition of arbitrary periodic function into its harmonic components is possible and unique.

Generally, \( \frac{1}{T}, \frac{2}{T}, \frac{4}{T}, \ldots \) components are also present, in this case missing due to the symmetry.
Fourier series

Decomposition of periodic $f(t)=f(t+T)$ function into the sum of harmonic functions

$$ f(t) = \sum_{n=-\infty}^{\infty} C_n e^{2\pi j n \nu_0 t}, \quad \nu_0 = \frac{1}{T} $$

$$ C_n = \frac{1}{T/2} \int_{t=-T/2}^{T/2} f(t) e^{-2\pi j n \nu_0 t} \, dt $$

alternative forms

$$ f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(2\pi n \nu_0 t\right) + b_n \sin\left(2\pi n \nu_0 t\right) \right] $$

$$ f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(2\pi n \nu_0 t + \phi_n\right) $$

decomposition of function $\cos(1.15x)$ in interval $<0,2\pi)$
black – original
green – sum to the 3rd harm.
red – cos, blue – sin (complex)

Fourier transformation

limit of Fourier series for $T \to \infty = \text{Fourier transformation}$

$$f(t) = \int_{-\infty}^{\infty} F(\nu) e^{2\pi j \nu t} \, d\nu \quad \text{and} \quad F(\nu) = \int_{-\infty}^{\infty} f(t) e^{-2\pi j \nu t} \, dt$$  
(is symmetric)
Fourier transformation examples

Gaussian pulse – smallest $\Delta t \Delta v$

infinite equidistant series of short pulses (with proper phase)
Fourier transformation properties & apps

properties

- linearity \( c_1 f_1(t) + c_2 f_2(t) \Leftrightarrow c_1 F_1(\nu) + c_2 F_2(\nu) \)
- shift \( f(t-c) \Leftrightarrow e^{-2\pi j c \nu} F(\nu) \)
- scaling \( f(ct) \Leftrightarrow \frac{F(\nu/c)}{|c|} \)
- derivative \( f'(t) \Leftrightarrow 2\pi j \nu F(\nu) \)
- convolution \( f_1(t) f_2(t) \Leftrightarrow F_1(\nu) * F_2(\nu) = \int_{-\infty}^{\infty} F_1(n) F_2(\nu-n) \, dn \)

FT applications

- sampling theorem
- harmonics analysis
- ideal linear refraction index frequency dependence
- ...

23
Laser pulse and spectrum

Laser pulse = \textit{time-limited output of continuous source}

- \text{monicromatic source}
- \text{periodic pulse source – discrete line spectrum}
- \text{common envelope is Gaussian}
- \text{periodic pulse source – discrete line spectrum}

\[
T_1 = 210 \text{ fs} \\
\nu_0 = 461 \text{ THz} \\
\lambda = 433 \text{ nm}, \Delta \lambda = 13 \text{ nm}
\]
Laser types by pulse width/spectrum

**continuous wave (CW)**

stable power and frequency, narrow bandwidth (line spectrum)

**chopped**

pulses ON - OFF, various pulse length, enables timing experiments

**pulsed**

short pulses (ps-fs), allow big peak energy, wide spectrum (continuous)

construction principles:
*Q – switching
*mode locking
Laser characteristics

spectral distribution

spatial modes

polarization

price

That's all.