BOSE-EINSTEIN CONDENSATION AND THE SUBMICROSCOPIC CONCEPT

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1 THE CONVENTIONAL VIEWPOINT

The possibility of Bose-Einstein condensation was obtained through the study of the behaviour of momentum states $\boldsymbol{k}$ of a gas of particles, i.e. by using methods of conventional quantum mechanics developed in an abstract phase space on the atomic scale, 10^{-10} m. As the density increases or the temperature decreases, the number of accessible states $\boldsymbol{k}$ per particle becomes smaller, such that at some moment more particles will be transferred into a single state $\boldsymbol{k}_0$ than statistical mechanics prescribes. Then single extra particles added to the system will go into that ground state too.

The Gross- Pitaevski equation

$$i\hbar\frac{\partial\psi(\mathbf{r})}{\partial t}=\left(-\frac{\hbar^2\nabla^2}{2m}+V(\mathbf{r})+U_0\,|\psi(\mathbf{r})|^2\right)\psi(\mathbf{r}).$$

is treated as the best equation describing the behaviour of the Bose-Einstein condensate and is often applied to analyse various aspects associated with the condensation.

2. THE STRUCTURE OF REAL SPACE [1]

We started from the necessity of the theorem about "something", which introduced preliminary elements needed to construct a physical universe. Only after that one may formulate a theorem about everything, i.e. the unification of all fundamental interactions, such as electromagnetic, weak, strong, and also ignoring equally important interactions such as gravitational and quantum mechanical (sometimes referred to as the Casimir force and similar phenomena).

The studies carried out in works show that the ordinary physical space is constrained to exist in the form of a tessellation lattice of primary topological balls. This mathematical lattice of balls was called the ***tessel-lattice***. The size of these balls, which in the tessel-lattice represent cells, was equated to the Planck length, $l_{\rm P}=\sqrt{\hbar G/c^3}\sim 10^{-35}$ m. A particle was determined as a local deformation of the tessel-lattice, i.e. a fractal volumetric contraction of a cell was associated with the creation of mass in the degenerate tessel-lattice. In particular, a stable fractal volumetric deformation was associated with a particle. Thus, an appearance of a local deformation in the tessel-lattice means the creation of matter in the degenerate space. A surface fractal deformation of such a particle means the creation of a charge and produces a charged particle.

3. THE SUBMICROSCOPIC MECHANICS [2]

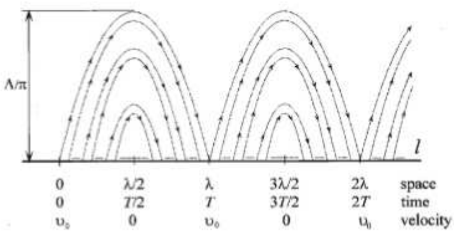
The motion of a particle was studied in a series of works by the author . It was shown that the motion of a particle is accompanied with a cloud of excitations, which were called ***inertons***, because these excitations are associated with the inert properties of matter.

The amplitude of the cloud of inertons (or the inerton cloud) is tied to the de Broglie wavelength by means of relationship

$$\Lambda=\lambda_{\rm de\ Br.}\,c/\upsilon \qquad (1)$$

where $\lambda_{\rm de\ Br.}$ is the spatial amplitude/period of the particle associated with the particle's de Broglie wavelength; c and υ are the velocity of light and the particle, respectively. The value of Λ in expression is the amplitude of the particle's inerton cloud, which spreads in transverse directions around the particle; along the particle's path it spreads up to the distance $\lambda_{\rm de\ Br.}/2$.

A moving particle jointly with its cloud of inertons represents the particle's wave ψ -function. The boundaries of the ψ-function are determined by the de Broglie wavelength and the amplitude of inertons. Thus, the ψ -function constitutes the field of inertia of the particle and inertons play the role of carriers of this field.



The behaviour of a canonical particle obeys submicroscopic

Results obtained by using submicroscopic mechanics allow us tolook behind the formalism of conventional probabilistic quantum mechanics developed on the atomic scale in an abstract phase space.

Acoustic excitations in condensed media, for example, phonons, have also to be surrounded by their inerton clouds. Since massive nodes vibrate near their equilibrium positions, i.e. are in motion, they emit and re-absorb clouds of inertons. Therefore inertons periodically remove a part of the mass from vibrating nodes and subsequently bring it back. Such behaviour can be described in terms of the Lagrangian

$$L_{\rm mass}=\sum_{\mathbf{n}}\left\{\tfrac{1}{2}\dot{m}_{\mathbf{n}}^2-\tfrac{\pi}{2\bar{T}}\left(\dot{m}_{\mathbf{n}}\mu_{\mathbf{n}}+\dot{m}_{\mathbf{n}+\mathbf{g}}\mu_{\mathbf{n}}\right)+\tfrac{1}{2}\dot{\mu}_{\mathbf{n}}^2\right\}$$

where $m_{\mathbf{n}}$ and $\mu_{\mathbf{n}}$ are variations of mass of **n**th node and its cloud of inertons, respectively, which occur due to the overlapping of inerton clouds of neighbouring nodes; $\boldsymbol{g}$ is the lattice vector; $\bar{T}$ is the period of collision of the mass located in the **n**th node with its inerton cloud. The dot over mass means the derivative in respect to the time *t* treated as a natural parameter.

4. CLUSTER FORMATION IN GASES

Overlapping inerton clouds of atoms in condensed media means that the appropriate term of interaction must be taken into account when one studies the equilibrium state of molecules. In other words, a pair potential of intermolecular interaction requires one more term associated with the interaction through the inerton channel. Thus the conventional consideration of a pair intermolecular interaction, for instance through the Lenard-Jones potential, must be supplement by an additional term: $V(r)=-\frac{\varepsilon_1}{r^6}+\frac{\varepsilon_2}{r^{12}}+\frac{1}{2}\gamma r^2$

where the last term represents elastic overlapping of inerton clouds of the nearest molecules. It seems the last term is small in solids and liquids where the electric components of interaction, the first and the second terms, prevail.

Let us consider an atomic gas. A decrease in temperature of the gas will also result in the decline of the de Broglie thermal wavelength $\lambda_{\rm th}=h/\sqrt{3mk_{\rm B}T}$ of gaseous atoms and at a certain temperature $\lambda_{\rm th}$ will become comparable to the mean distance g between atoms. In terms of conventional quantum mechanical consideration this means that the atomic wave functions start to overlap and the atoms by Ketterle "become a 'quantum soup' of indistinguishable particles". In such a "soup" atoms are found in the coherent state.

However, submicroscopic mechanics [2] states that the overlapping of inerton clouds of entities takes place much earlier at a higher temperature and a larger distance between atoms, because the overlapping is determined by relationship (1). For example, in a solid the wavelength of an atom is around 10^{-11} m and the velocity of motion/vibration is around 10^3 m/s, then $\Lambda\sim10^{-6}$, which is a quite big distance in the context of condensed matter. This overlapping gives rise to the appearance of collective vibrations known as phonons.

From the view point of the sub microscopic concept the whole coherent state in which the motion of all the atoms is synchronised requires other conditions, namely, when the de Broglie thermal wavelength $\lambda_{\rm th}$ becomes exactly equal to the distance g between atoms. But what happens in this case? In this case the $(\mathbf{n}-\mathbf{g})$th atom emits its inerton cloud that then is fully absorbed by the **n**th atom; the **n**th atom emits its own cloud of inertons, which then is fully absorbed by the $(\mathbf{n}+\mathbf{g})$th atom, etc. In other words, the coherent exchange of inerton clouds by the atoms when an inerton cloud emitted by one atom hops to the neighbouring atom and is absorbed by it, we have to relate with the phenomenon of Bose-Einstein condensation.

Let us investigate whether the occurrence of clusters can be possible in a Bose-Einstein condensate. The action S for the ensemble of $N_{\rm total}$ interacting boson particles, which tend to clusterise with N particles in a cluster, has the form [3]

$$S\cong K\cdot\left\{\tfrac{1}{2}(F_{\rm rep}-F_{\rm att})N^2-\ln(N+1)\right\} \qquad (2)$$

where the functions $F_{\rm rep/att}$ are determined as

$$F_{\rm rep/att}(N)=(3/k_{\rm B}T)\int_1^{N^{1/3}}V_{\rm rep/att}(\rho)\rho^2 d\rho\,,$$

For simplicity, the repulsion potential can be taken in the Lenard-Jones' form

$$V_{\rm rep}(\rho)=V_{0\ \rm rep}/\rho^{12}.$$

The attraction potential should include: 1) the dispersion potential of inter-atomic interaction $-C_6/r^6$; 2) a potential formed by a trap, which can be modeled by a harmonic potential and 3) the harmonic potential caused by small spatial oscillations of atoms near their equilibrium positions, i.e. inerton elastic interaction. So, the attraction potential is

$$V_{\rm att}(\rho)=C_6/(r\rho)^6-\tfrac{1}{2}\gamma_{\rm trap}\,r^2\rho^2-\tfrac{1}{2}m\omega^2(\delta r)^2\rho^2$$

where m is the mass of an atom, r is the distance between atoms, γ is the effective force constant of the trap, ω is the cyclic frequency of proper oscillations of an atom, δr is the appropriate amplitude and ρ is the dimensionless distance parameter.

Substituting potentials $V_{\rm rep/att}$ into functions $F_{\rm rep/att}$ we obtain instead of expression (2) the action

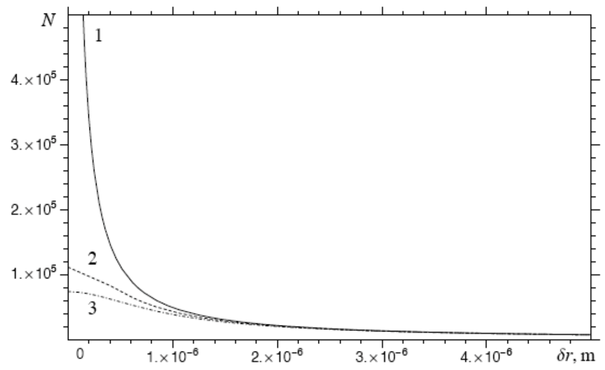
$$S=K\cdot\left\{\tfrac{1}{6}\frac{V_{0\ \rm rep}}{k_{\rm B}T}N^2-\tfrac{3}{6}\frac{C_6}{r^6k_{\rm B}T}N^2+\tfrac{3}{20}\frac{1}{k_{\rm B}T}\left(\gamma_{\rm trap}r^2+m\omega^2\delta^2\right)N^{11/3}-\ln(N-1)\right\}$$

Proper oscillations of atoms, which are characterised by the frequency ω, are produced by their movements. In other words, the origin of the frequency ω is produced by collisions of atoms with their inerton clouds.

The number of atoms in a cluster, $\partial S/\partial N=0$:

$$N\cong\left(\frac{10}{33}\,\frac{9C_6/r^6-4V_{0\ \rm rep}}{\gamma_{\rm trap}\,r^2+m\omega^2\delta r^2}\right)^{3/5}. \qquad (3)$$

A variation of the parameters in expression (3) allows us to construct the dependence of the number of atoms in a cluster N versus the de Broglie thermal wavelength, or amplitude δr; these estimates are in line with experimental observations that show 10^5 up to around 10^{10} atoms being in the state of Bose-Einstein condensation:



Number of atoms N in a Bose-Einstein condensate cluster versus amplitude δr of thermal motion of atoms, i.e. the solution of Eq. (20) as the function of the de Broglie thermal wavelength $\delta r=\lambda_{\rm th}=h/(m_{\rm Cs}\upsilon_{\rm th})=h/\sqrt{3k_{\rm B}T\,m_{\rm Cs}}$. Here, the double harmonic trapping potential $\gamma_{\rm trap}\,r^2$ = 0 (curve 1), 5×10^{-29} J (curve 2) and 5×10^{-28} J (curve 3).

Thus, the presented consideration makes it possible to account for the quantity of matter that is found in the condensed state: inter-atomic interactions subdivide the system of atoms to clusters but the cluster state is realized when the absolute value of an attraction potential starts to exceeds the thermal energy, $V_{\rm att}\ge k_{\rm B}T$. This inequality holds for the case calculated above: the attraction energy $\tfrac{1}{2}m_{\rm Cs}\,\omega^2\delta r^2\approx1.7\times10^{-30}$ J exceeds the thermal energy $k_{\rm B}T\approx7\times10^{-31}$ J.

REFERENCES

[1] M. Bounias and V. Krasnoholovets, Scanning the structure of ill-known spaces: Part 1,2,3. *Kybernetes: The Int. J. Systems and Cybernetics* **32**, no. 7/8, 945-1020 (2003) http://arXiv.org/abs/physics/0211096 http://arXiv.org/abs/physics/0212004 http://arXiv.org/abs/physics/0301049

[2] V. Krasnoholovets, Submicroscopic deterministic quantum mechanics, *Int. J. Computing Anticipatory Systems* **11**, 164-179 (2002) http://arXiv.org/abs/quant-ph/0109012

[3] V. Krasnoholovets and B. Lev, Systems of particles with interaction and the cluster formation in condensed matter, *Cond. Matt. Phys.* **6**, no. 1, 67-83 (2003) http://arXiv.org/abs/cond-mat/0210131